Dionysius Exiguus, On Easter, or, the Paschal Cycle (2003)

Preface

Dionysius Exiguus to the most blessed and very dear father, Petronius, bishop.

The reasoning of the feast of Easter, which many have frequently and urgently asked us, with the help of your prayers, we have now proceeded to set forth. Following in all things the venerable 318 pontiffs, who came together at Nicaea, a city of Bithynia, against the madness of Arius, and besides [gave] a perfect and true opinion on this matter; who having observed 14 months of Easter through 19 years always returning in a cycle to the same position, fixed it stable and immoveable, which in all ages is repeated in the same way, as a beginning, without going off into an excursion of various things. However they sanctioned this rule of the aforementioned cycle, not so much from secular knowledge as by the illumination of the Holy Spirit, and as if determined to have assigned a firm and stable anchor to this reasoning of the lunar calculation. As after a while some, whether despising from arrogance or crossing over from ignorance, were influenced by Jewish fables, they handed down a different and contrary form of the only festival. And because without solidity of foundation no structure can stand, for a long time they were inclined to work out differently the Lord's Easter and the computation of the moon, ordaining unordained cycles; which not only has no stability, indeed also they prefer a notable direction in error.

But at the city of Alexandria the archbishop, blessed Athanasius, who also was involved in the Nicene Council, at that time as deacon of the holy pontiff Alexander and [his] helper in all things, and then the venerable Theophilus and Cyril, departed very little from the worshipful decision of the synod. Indeed rather sollicitously retaining the same 19-year cycle, which in a Greek word is called *enneacaidecaeteris*, they are shown to have not interpolated the paschal cycle with any changes. Then Pope Theophilus, dedicating the 100th course of the years to the emperor Theodosius the Elder, and St. Cyril, compiling a cycle of time of 95 years, preserved through everything this tradition of the holy council of the importance of observing 14 paschal months. And because -- the students also having been seeking to know what is true -- we must hold fast to the rule of his cycle more firmly, we believe that we must give it after our preface.

Therefore we hurried to set out this cycle of 95 years, in which study we have succeeded, preferring in our work this [cycle], the last one of the same blessed Cyril, that is the 5th cycle, because 6 years of it remain; and thereafter we profess that we laid out 5 others according to the pattern of the same pontiff, or rather of the often mentioned Nicene Council. But because St. Cyril began his first cycle from the 153rd year of Diocletian, and besides ended in the 247th, we, starting from the 248th [year] of the same tyrant -- a better [word] than prince -- do not wish to bind to our circles the memory of this impious man and persecutor, but choose rather to count the time of the years from the incarnation of our Lord Jesus Christ, so that the beginning of our hope will appear better known to us, and the cause of the restoration of mankind, i.e. the passion of our Redeemer, may shine forth more clearly.

In addition we think that this reader should be reminded that that cycle of 95 years, which we make, when, its time being up, begins to repeat, **not through everything may he support firmness**.

For it is allowed ... the years of our Lord Jesus Christ ... his/its order ... for the continued series .... that they may preserve ... , and they might run through the accustomed indictions through 15 years, also the epacts, as the Greeks call them, i.e the additions ... 11 annual months ... which 30 of days up to in itself they return, ...

...however they are unable to protect a similar movement of constancy concurrent days of the week, and the day(s) of the Lord's Pasch and the month of the dominical day itself. However the reason of the concurrency of the week, which comes from the course of the sun, it is concluded in a continual circuit of seven years. In which you will take care to enumerate through the years each one; only in that year in which it will have been a leap year, you will add two. Which cause also makes that not through the whole 95 years does that circle seem to harmonize with its recursion. For when in other years it does not deviate, in this alone, in which the leap year is inserted, the Pasch of the churches with its month occurs in various ways of reason (rationis). ....

... (To be completed)

End of the preface

NINETEEN YEAR CYCLE OF DIONYSIUS (CYCLUS DECEMNOVENNALIS DIONYSII)

[Translated by Michael Deckers]

The following translation is as literal as I could do it in order to reflect the style and diction of the original. Subsequent comments contain a translation of the calendrical content into modern algebraic notation.

The Latin text has been transcribed and edited by Rodolphe Audette ("http://hermes.ulaval.ca/~sitrau/calgreg/denys.html").

I have profited from the learned comments and helpful suggestions by Joe Kress, A R Tom Peters, and Robert H van Gent. Any typos in the Latin text and errors in the translation and the comments are mine. [http://the-light.com/cal/DionysiusArgumenta.txt]

CYCLUS DECEMNOVENNALIS DIONYSII	NINETEEN YEAR CYCLE OF DIONYSIUS

Incipit cyclus decemnovennalis, quem Graeci Enneacaidecaeterida vocant, constitutus a sanctis Patribus, in quo quartas decimas paschales omni tempore sine ulla reperies falsitate; tantum memineris annis singulis, qui cyclus lunae et qui decemnovennalis existat. In praesenti namque tertia indictio est, consulatu Probi iunioris, tertius decimus circulus decemnovennalis, decimus lunaris est.	The nineteen year cycle begins, which the Greek call Enneakaidekaeterida (nineteen yearly), established by the holy [Church] Fathers, in which you shall find fourteen paschal[moon]s each time without error; you shall just bear in mind, in each of the years, which cycle of the moon and which nineteen year [cycle] prevails. In the present [year], in the consulship of Probus Junior, it is the thirteenth of the nineteen year cycle, and the tenth lunar one.

ANNI DIOCLE TIANI	**quae sint indictiones**	**epactae, id est adjectiones lunae**	**concurrentes dies**	**quotus sit lunae circsulus**	**quae sit luna XIIII paschalis**	**dies Dominicae festivitatis**	**quota sit luna ipsius diei dominici**
YARS OF DIOCLETIAN	What are the indictions	epacts, ie increments of the moon	concurrent days	which is the circle of the moon	date of day 14 of the paschal moon	day of the Sunday festival	which is the day of the moon on this Sunday
CCXXVIIII 229 (513)	**vi**	**nulla**	**i**	**xvii**	**non.Apr.** Apr 05	**vii id.Apr.** Apr 07	**xvi**
CCXXX 230 (514)	**vii**	**xi**	**ii**	**xviii**	**viii k.Apr.** Mar 25	**iii k.Apr.** Mar 30	**xviiii**

CCXXXI 231 (515)	**viii**	**xxii**	**iii**	**xviiii**	**id. Apr.** Apr 13	**xiii k.Maii** Apr 19	**xx**
CCXXXII 232 (516)	**viiii**	**iii**	**v**	**i**	**non.Apr.** Apr 02	**iii non.Apr.** Apr 03	**xv**
CCXXXIII 233 (517)	**x**	**xiiii**	**vi**	**ii**	**xi k.Apr.** Mar 22	**vii k.Apr.** Mar 26	**xviii**
CCXXXIIII 234 (518)	**xi**	**xxv**	**vi**	**iii**	**iiii id.Apr.** Apr 10	**xvii k.Maii** Apr 15	**xviiii**
CCXXXV 235 (519)	**xii**	**vi**	**i**	**iiii**	**iii k.Apr.** Mar 30	**ii k.Apr.** Mar 31	**xv** *
CCXXXVI 236 (520)	**xiii**	**xvii**	**iii**	**v**	**xiiii k.Maii** Apr 18*	**xiii k.Maii** Apr 19	**xv ogd.** *
CCXXXVII 237 (521)	**xiiii**	**xxviii**	**iiii**	**vi**	**vii id.Apr.** Apr 07	**iii id.Apr.** Apr 11	**xviii**
CCXXXVIII 238 (522)	**xv**	**viiii**	**v**	**vii**	**vi k.Apr.** Mar 27	**iii non.Apr.** Apr 03	**xxi** *
CCXXXVIIII 239 (523)	**i**	**xx**	**vi**	**viii**	**xvii k.Maii** Apr 15	**xvi k.Maii** Apr 16	**xv** *
CCXL 240 (524)	**ii**	**i**	**i**	**viiii**	**ii non.Apr.** Apr 04	**vii id.Apr.** Apr 07	**xvii**
CCXLI 241 (525)	**iii**	**xii**	**ii**	**x**	**viiii k.Apr.** Mar 24	**iii k.Apr.** Mar 30	**xx**
CCXLII 242 (526)	**iiii**	**xxiii**	**iii**	**xi**	**ii id.Apr.** Apr 12	**xiii k.Maii** Apr 19	**xxi** *
CCXLIII 243 (527)	**v**	**iiii**	**iiii**	**xii**	**k.Apr.** Apr 01	**ii non.Apr.** Apr 04	**xvii**
CCXLIIII 244 (528)	**vi**	**xv**	**vi**	**xiii**	**xii k.Apr.** Mar 21*	**vii k.Apr.** Mar 26	**xviiii**
CCXLV 245 (529)	**vii**	**xxvi**	**vii**	**xiiii**	**v id.Apr.** Apr 09	**xvii k.Maii** Apr 15	**xx**
CCXLVI 246 (530)	**viii**	**vii**	**i**	**xv**	**iiii k.Apr.** Mar 29	**ii k.Apr.** Mar 31	**xvi**
CCXLVII 247 (531)	**viiii**	**xviii**	**ii**	**xvi**	**xv k.Maii** Apr 17	**xii k.Maii** Apr 20	**xvii hend.**

ANNI DOMINI NOSTRI JESU CHRISTI	**quae sint indictiones**	**epactae, id est adjectiones lunae**	**concurrentes dies**	**quotus sit lunae circsulus**	**quae sit luna XIIII paschalis**	**dies Dominicae festivitatis**	**quota sit luna ipsius diei dominici**
YARS OF OUR LORD JESUS CHRIST	What are the indictions	epacts, ie increments of the moon	concurrent days	which is the circle of the moon	date of day 14 of the paschal moon	day of the Sunday festival	which is the day of the moon on this Sunday
B DXXXII 0532	**x**	**nulla**	**iiii**	**xvii**	**non.Apr.** Apr 05	**iii id.Apr.** Apr 11	**xx**
DXXXIII 0533	**xi**	**xi**	**v**	**xviii**	**viii k.Apr.** Mar 25	**vi k.Apr.** Mar 27	**xvi**

DXXXIIII 0534	**xii**	**xxii**	**vi**	**xviiii**	**id.Apr.** Apr 13	**xvi k.Maii** Apr 16	**xvii**
DXXXV 0535	**xiii**	**iii**	**vii**	**i**	**iiii non.Apr.** Apr 02	**vi id.Apr.** Apr 08	**xx**
B DXXXVI 0536	**xiiii**	**xiiii**	**ii**	**ii**	**xi k.Apr.** Mar 22	**x k.Apr.** Mar 23	**xv** *
DXXXVII 0537	**xv**	**xxv**	**iii**	**iii**	**iiii id.Apr.** Apr 10	**ii id.Apr.** Apr 12	**xvi**
DXXXVIII 0538	**i**	**vi**	**iiii**	**iiii**	**iii k.Apr.** Mar 30	**ii non.Apr.** Apr 04	**xviiii**
DXXXVIIII 0539	**ii**	**xvii**	**v**	**v**	**xiiii k.Maii** Apr 18*	**viii k.Maii** Apr 24	**xx ogd.**
B DXL 0540	**iii**	**xxviii**	**vii**	**vi**	**vii id.Apr.** Apr 07	**vi id.Apr.** Apr 08	**xv** *
DXLI 0541	**iiii**	**viiii**	**i**	**vii**	**vi k.Apr.** Mar 27	**ii k.Apr.** Mar 31	**xviii**
DXLII 0542	**v**	**xx**	**ii**	**viii**	**xvii k.Maii** Apr 15	**xii k.Maii** Apr 20	**xviiii**
DXLIII 0543	**vi**	**i**	**iii**	**viiii**	**ii non.Apr.** Apr 04	**non.Apr.** Apr 05	**xv** *
B DXLIIII 0544	**vii**	**xii**	**v**	**x**	**viiii k.Apr.** Mar 24	**vi k.Apr.** Mar 27	**xvii**
DXLV 0545	**viii**	**xxiii**	**vi**	**xi**	**ii id.Apr.** Apr 12	**xvi k.Maii** Apr 16	**xviii**
DXLVI 0546	**viiii**	**iiii**	**vii**	**xii**	**k.Apr.** Apr 01	**vi id.Apr.** Apr 08	**xxi** *
DXLVII 0547	**x**	**xv**	**i**	**xiii**	**xii k.Apr.** Mar 21*	**viiii k.Apr.** Mar 24	**xvii**
B DXLVIII 0548	**xi**	**xxvi**	**iii**	**xiiii**	**v id.Apr.** Apr 09	**ii id.Apr.** Apr 12	**xvii**
DXLVIIII 0549	**xii**	**vii**	**iiii**	**xv**	**iiii k.Apr.** Mar 29	**ii non.Apr.** Apr 04	**xx**
DL 0550	**xiii**	**xviii**	**v**	**xvi**	**xv k.Maii** Apr 17	**viii k.Maii** Apr 24	**xxi hend.** *
DLI 0551	**xiiii**	**nulla**	**vi**	**xvii**	**non.Apr.** Apr 05	**v id.Apr.** Apr 09	**xviii**
B DLII 0552	**xv**	**xi**	**i**	**xviii**	**viii k.Apr.** Mar 25	**ii k.Apr.** Mar 31	**xx**
DLIII 0553	**i**	**xxii**	**ii**	**xviiii**	**id.Apr.** Apr 13	**xii k.Maii** Apr 20	**xxi** *
DLIIII 0554	**ii**	**iii**	**iii**	**i**	**iiii non.Apr.** Apr 02	**non.Apr.** Apr 05	**xvii**
DLV 0555	**iii**	**xiiii**	**iiii**	**ii**	**xi k.Apr.** Mar 22	**v k.Apr.** Mar 28	**xx**
B DLVI 0556	**iiii**	**xxv**	**vi**	**iii**	**iiii id.Apr.** Apr 10	**xvi k.Maii** Apr 16	**xx**
DLVII 0557	**v**	**vi**	**vii**	**iiii**	**iii k.Apr.** Mar 30	**k.Apr.** Apr 01	**xvi**

DLVIII 0558	**vi**	**xvii**	**i**	**v**	**xiiii k.Maii** Apr 18*	**xi k.Maii** Apr 21	**xvii ogd.**
DLVIIII 0559	**vii**	**xxviii**	**ii**	**vi**	**vii id.Apr.** Apr 07	**id.Apr.** Apr 13	**xx**
B DLX 0560	**viii**	**viiii**	**iiii**	**vii**	**vi k.Apr.** Mar 27	**v k.Apr.** Mar 28	**xv** *
DLXI 0561	**viiii**	**xx**	**v**	**viii**	**xvii k.Maii** Apr 15	**xv k.Maii** Apr 17	**xvi**
DLXII 0562	**x**	**i**	**vi**	**viiii**	**ii non.Apr.** Apr 04	**v id.Apr.** Apr 09	**xviiii**
DLXIII 0563	**xi**	**xii**	**vii**	**x**	**viiii k.Apr.** Mar 24	**viii k.Apr.** Mar 25	**xv** *
B DLXIIII 0564	**xii**	**xxiii**	**ii**	**xi**	**ii id.Apr.** Apr 12	**id.Apr.** Apr 13	**xv** *
DLXV 0565	**xiii**	**iiii**	**iii**	**xii**	**k.Apr.** Apr 01	**non.Apr.** Apr 05	**xviii**
DLXVI 0566	**xiiii**	**xv**	**iiii**	**xiii**	**xii k.Apr.** Mar 21*	**v k.Apr.** Mar 28	**xxi** *
DLXVII 0567	**xv**	**xxvi**	**v**	**xiiii**	**v id.Apr.** Apr 09	**iiii id.Apr.** Apr 10	**xv** *
B DLXVIII 0568	**i**	**vii**	**vii**	**xv**	**iiii k.Apr.** Mar 29	**k.Apr.** Apr 01	**xii**
DLXVIIII 0569	**ii**	**xviii**	**i**	**xvi**	**xv k.Maii** Apr 17	**xi k.Maii** Apr 21	**xviii hend.**
DLXX 0570	**iii**	**nulla**	**ii**	**xvii**	**non.Apr.** Apr 05	**viii id.Apr.** Apr 06	**xv** *
DLXXI 0571	**iiii**	**xi**	**iii**	**xviii**	**viii k.Apr.** Mar 25	**iiii k.Apr.** Mar 29	**xviii**
B DLXXII 0572	**v**	**xxii**	**v**	**xviiii**	**id.Apr.** Apr 13	**xv k.Maii** Apr 17	**xviii**
DLXXIII 0573	**vi**	**iii**	**vi**	**i**	**iiii non.Apr.** Apr 02	**v id.Apr.** Apr 09	**xxi** *
DLXXIIII 0574	**vii**	**xiiii**	**vii**	**ii**	**xi k.Apr.** Mar 22	**viii k.Apr.** Mar 25	**xvii**
DLXXV 0575	**viii**	**xxv**	**i**	**iii**	**iiii id.Apr.** Apr 10	**xviii k.Maii** Apr 14	**xviii**
B DLXXVI 0576	**viiii**	**vi**	**iii**	**iiii**	**iii k.Apr.** Mar 30	**non.Apr.** Apr 05	**xx**
DLXXVII 0577	**x**	**xvii**	**iiii**	**v**	**xiiii k.Maii** Apr 18*	**vii k.Maii** Apr 25*	**xxi ogd.** *
DLXXVIII 0578	**xi**	**xxviii**	**v**	**vi**	**vii id.Apr.** Apr 07	**iiii id.Apr** Apr 10	**xvii**
DLXXVIIII 0579	**xii**	**viiii**	**vi**	**vii**	**vi k.Apr.** Mar 27	**iiii non.Apr.** Apr 02	**xx**
B DLXXX 0580	**xiii**	**xx**	**i**	**viii**	**xvii k.Maii** Apr 15	**xi k.Maii** Apr 21	**xx**
DLXXXI 0581	**xiiii**	**i**	**ii**	**viiii**	**ii non.Apr.** Apr 04	**viii id.Apr.** Apr 06	**xvi**

DLXXXII 0582	**xv**	**xii**	**iii**	**x**	**viiii k.Apr.** Mar 24	**iiii k.Apr.** Mar 29	**xviiii**
DLXXXIII 0583	**i**	**xxiii**	**iiii**	**xi**	**ii id.Apr.** Apr 12	**xiiii k.Maii** Apr 18	**xx**
B DLXXXIIII 0584	**ii**	**iiii**	**vi**	**xii**	**k.Apr.** Apr 01	**iiii non.Apr.** Apr 02	**xv** *
DLXXXV 0585	**iii**	**xv**	**vii**	**xiii**	**xii k.Apr.** Mar 21*	**viii k.Apr.** Mar 25	**xviii**
DLXXXVI 0586	**iiii**	**xxvi**	**i**	**xiiii**	**v id.Apr.** Apr 09	**xviii k.Maii** Apr 14	**xviiii**
DLXXXVII 0587	**v**	**vii**	**ii**	**xv**	**iiii k.Apr.** Mar 29	**iii k.Apr.** Mar 30	**xv** *
B DLXXXVIII 0588	**vi**	**xviii**	**iiii**	**xvi**	**xv k.Maii** Apr 17	**xiiii k.Maii** Apr 18	**xv hend.** *
DLXXXVIIII 0589	**vii**	**nulla**	**v**	**xvii**	**non.Apr.** Apr 05	**iiii id.Apr.** Apr 10	**xviiii**
DXC 0590	**viii**	**xi**	**vi**	**xviii**	**viii k.Apr.** Mar 25	**vii k.Apr.** Mar 26	**xv** *
DXCI 0591	**viiii**	**xxii**	**vii**	**xviiii**	**id.Apr.** Apr 13	**xvii k.Maii** Apr 15	**xvi**
B DXCII 0592	**x**	**iii**	**ii**	**i**	**iiii non.Apr.** Apr 02	**viii id.Apr.** Apr 06	**xviii**
DXCIII 0593	**xi**	**xiiii**	**iii**	**ii**	**xi k.Apr.** Mar 22	**iiii k.Apr.** Mar 29	**xxi** *
DXCIIII 0594	**xii**	**xxv**	**iiii**	**iii**	**iiii id.Apr.** Apr 10	**iii id.Apr.** Apr 11	**xv** *
DXCV 0595	**xiii**	**vi**	**v**	**iiii**	**iii k.Apr.** Mar 30	**iii non.Apr.** Apr 03	**xviii**
B DXCVI 0596	**xiiii**	**xvii**	**vii**	**v**	**xiiii k.Maii** Apr 18*	**x k.Maii** Apr 22	**xviii ogd.**
DXCVII 0597	**xv**	**xxviii**	**i**	**vi**	**vii id.Apr.** Apr 07	**xviii k.Maii** Apr 14	**xxi** *
DXCVIII 0598	**i**	**viiii**	**ii**	**vii**	**vi k.Apr.** Mar 27	**iii k.Apr.** Mar 30	**xvii**
DXCVIIII 0599	**ii**	**xx**	**iii**	**viii**	**xvii k.Maii** Apr 15	**xiii k.Maii** Apr 19	**xviii**
B DC 0600	**iii**	**i**	**v**	**viiii**	**ii non.Apr.** Apr 04	**iiii id.Apr.** Apr 10	**xx**
DCI 0601	**iiii**	**xii**	**vi**	**x**	**viiii k.Apr.** Mar 24	**vii k.Apr.** Mar 26	**xvi**
DCII 0602	**v**	**xxiii**	**vii**	**xi**	**ii id.Apr.** Apr 12	**xvii k.Maii** Apr 15	**xvii**
DCIII 0603	**vi**	**iiii**	**i**	**xii**	**k.Apr.** Apr 01	**vii id.Apr.** Apr 07	**xx**
B DCIIII 0604	**vii**	**xv**	**iii**	**xiii**	**xii k.Apr.** Mar 21*	**xi k.Apr.** Mar 22*	**xv** *

DCV 0605	**viii**	**xxvi**	**iiii**	**xiiii**	**v id.Apr.** Apr 09	**iii id.Apr.** Apr 11	**xvi**
DCVI 0606	**viiii**	**vii**	**v**	**xv**	**iiii k.Apr.** Mar 29	**iii non.Apr.** Apr 03	**xviiii**
DCVII 0607	**x**	**xviii**	**vi**	**xvi**	**xv k.Maii** Apr 17	**viiii k.Maii** Apr 23	**xx hend.**
B DCVIII 0608	**xi**	**nulla**	**i**	**xvii**	**non.Apr.** Apr 05	**vii id.Apr.** Apr 07	**xvi**
DCVIIII 0609	**xii**	**xi**	**ii**	**xviii**	**viii k.Apr.** Mar 25	**iii k.Apr.** Mar 30	**xviiii**
DCX 0610	**xiii**	**xxii**	**iii**	**xviiii**	**id.Apr.** Apr 13	**xiii k.Maii** Apr 19	**xx**
DCXI 0611	**xiiii**	**iii**	**iiii**	**i**	**iiii non.Apr.** Apr 02	**ii non.Apr.** Apr 04	**xvi**
B DCXII 0612	**xv**	**xiiii**	**vi**	**ii**	**xi k.Apr.** Mar 22	**vii k.Apr.** Mar 26	**xviii**
DCXIII 0613	**i**	**xxv**	**vii**	**iii**	**iiii id.Apr.** Apr 10	**xvii k.Maii** Apr 15	**xviiii**
DCXIIII 0614	**ii**	**vi**	**i**	**iiii**	**iii k.Apr.** Mar 30	**ii k.Apr.** Mar 31	**xv** *
DCXV 0615	**iii**	**xvii**	**ii**	**v**	**xiiii k.Maii** Apr 18*	**xii k.Maii** Apr 20	**xvi ogd.**
B DCXVI 0616	**iiii**	**xxviii**	**iiii**	**vi**	**vii id.Apr.** Apr 07	**iii id.Apr.** Apr 11	**xviii**
DCXVII 0617	**v**	**viiii**	**v**	**vii**	**vi k.Apr.** Mar 27	**iii non.Apr.** Apr 03	**xxi** *
DCXVIII 0618	**vi**	**xx**	**vi**	**viii**	**xvii k.Maii** Apr 15	**xvi k.Maii** Apr 16	**xv** *
DCXVIIII 0619	**vii**	**i**	**vii**	**viiii**	**ii non.Apr.** Apr 04	**vi id.Apr.** Apr 08	**xviii**
B DCXX 0620	**viii**	**xii**	**ii**	**x**	**viiii k.Apr.** Mar 24	**iii k.Apr.** Mar 30	**xx**
DCXXI 0621	**viiii**	**xxiii**	**iii**	**xi**	**ii id.Apr.** Apr 12	**xiii k.Maii** Apr 19	**xxi** *
DCXXII 0622	**x**	**iiii**	**iiii**	**xii**	**k.Apr.** Apr 01	**ii non.Apr.** Apr 04	**xvii**
DCXXIII 0623	**xi**	**xv**	**v**	**xiii**	**xii k.Apr.** Mar 21*	**vi k.Apr.** Mar 27	**xx**
B DCXXIIII 0624	**xii**	**xxvi**	**vii**	**xiiii**	**v id.Apr.** Apr 09	**xvii k.Maii** Apr 15	**xx**
DCXXV 0625	**xiii**	**vii**	**i**	**xv**	**iiii k.Apr.** Mar 29	**ii k.Apr.** Mar 31	**xvi**
DCXXVI 0626	**xiiii**	**xviii**	**ii**	**xvi**	**xv k.Maii** Apr 17	**xii k.Maii** Apr 20	**xvii hend.**

The leftmost column in this table gives a year number pertaining to the feast of Easter described in each line, and to the indiction:
column 1 =(in the second part of the table:) the year number Y of the incarnation =(in the first part of the table:) the Diocletian year number D = Y - 284

Column 1 has the prefix "B" (for bissextum) if Y mod 4 = D mod 4 = 0.

The other columns are all related to Y, as follows:
*column 2 = 1 + (2 + Y)mod 15 (see [Argumentum 2]) column 3 = ((Y mod 19)*11) mod 30 (see [Argumentum 3]) column 4 = 1 + (3 + floor(Y*5/4))mod 7 (see [Argumentum 4]) column 5 = 1 + (Y - 3)mod 19 (see [Argumentum 6]) column 6 = March 21 + (15 - 11*(Y mod 19))mod 30 d (see [Argumentum 14])*

Column 8 has the postfix "ogd." (for ogdoadas) if Y mod 19 = 8 - 1 and the postfix "hend." (for hendeka) if Y mod 19 = 11 + 8 - 1.

We have indicated extreme values in columns 6, 7, 8 with asterisks.

ARGUMENTA PASCHALIA

Incipiunt argumenta de titulis paschalibus ¦gyptiorum investigata solertia ut praesentes indicent.	This begins the argumenta on the determination of Easter by the Egyptians, carefully investigated as shown in the following.
<u>Argumentum primum. De annis Christi.</u> **Si nosse vis quotus sit annus ab incarnatione Domini nostri Jesu Christi, computa quindecies XXXIV, fiunt DX; iis semper adde XII regulares, fiunt DXXII; adde etiam indictionem anni cujus volueris, ut puta, tertiam, consulatu Probi junioris, fiunt simul anni DXXV. Isti sunt anni ab incarnatione Domini.**	<u>First Argumentum. On the years of Christ.</u> If you want to find out which year it is since the incarnation of our Lord Jesus Christ, compute fifteen times 34, yielding 510; to these always add the correction 12, yielding 522; also add the indiction of the year you want, say, in the consulship of Probus Junior, the third, yielding 525 years altogether. These are the years since the incarnation of the Lord.

That the year numbers Y employed here agree with the usual Julian year numbers J is shown by the formulae for the indiction in [Argumentum 2] (using Y mod 15), for the epacts in [Argumenta 3 and 11] (using Y mod 19), and for the day of the week in [Argumentum 4] (using Y mod 28).

It is not clear from the text, however, when the years of the incarnation are supposed to begin. The formulae imply that Y and J agree on January 01 ([Argumentum 12]), on the leap day ([Argumentum 8]), and around Easter; and [Argumentum 2] suggests that Y and J agree until September 01. The year 0001 of the era of Diocletianus, which Dionysius wants to replace with era domini, is usually taken to start at the sunset epoch Julian date(0284, August, 28.75).

Some of the assertions on the birthday of Jesus in [Argumentum 15] below would render "anni ab incarnatione Domini" a misnomer.

<u>Argumentum II. De indictione.</u> **Si vis scire quota est indictio, ut puta, consulatu Probi junioris, sume annos ab incarnatione Domini nostri Jesu Christi DXXV. His semper adjice III, fiunt DXXVIII. Hos partire per XV, remanent III. Tertia est indictio. Si vero nihil remanserit, decima quinta indictio est.**	<u>Argumentum 2.</u> On the indiction. If you want to know which indiction it is, say in the consulship of Probus Junior, then add the years since the incarnation of our Lord Jesus Christ, 525. To this always add 3, yielding 528. Divide these by 15, 3 are left over. It is the third indiction. But if nothing would be left over then it is the fifteenth indiction.

For year number Y this gives indiction(Y) = 1 + (2 + Y)mod 15 as is confirmed by the second column in the table above. This is in fact the cycle of indiction number for Julian year Y from January 01 until that number changes later in the year (on September 01 or some time later).

Argumentum III. De epactis.	Argumentum 3. On the epacts.
Si vis cognoscere quot sint epactae, id est adjectiones lunares, sume annos ab incarnatione Domini nostri Jesu Christi, quot fuerint DXXV. Hos partire per XIX, remanent XII. Per XI multiplica, fiunt CXXXII. Hos item partire per XXX, remanent XII. Duodecim sunt adjectiones lunares.	If you want to learn the number of epacts, that is, of the lunar increments, then add the years since the incarnation of our Lord Jesus Christ, of which 525 have passed. Divide those by 19, 12 are left over. Multiply by 11, yielding 132. And then divide those by 30, 12 are left over. Twelve is the lunar increment.

*For year number Y this is meant to describe the formula epacts(Y) = ((Y mod 19)*11) mod 30 (where both modulo operations can yield zero) as is confirmed by the third column in the table above. Since Y is integral, this is the same as epacts(Y) = floor(Y*(235/19)*30) mod 30 which shows that the formula uses the Metonic value of (calendar year)/(synodic month) ~= 235/19 ~= (365.25 d)/(29.530 85 d). This estimate of the synodic month exceeds modern estimates by only 1 d in about 300 years.*

*The formula for the epacts does in fact extend the epacts given for the Diocletian year numbers D = Y - 284 in the table above. Note that (Y - D) mod 19 = 18, which makes the formula for Y somewhat simpler to express verbally than that for D (because no "regulares" are needed): epacts for Diocletian year number(D) = ((D - 1)mod 19)*11) mod 30.*

The formula for the epacts remains the same if the year number Y is replaced by the year number S = Y + 38 since the Spanish era (this count may have been known to Dionysius).

Argumentum IV. De concurrentibus.	Argumentum 4. On the concurrents.
Si vis scire adjectiones solis, id est concurrentes septimanae dies, sume annos ab incarnatione Domini quot fuerint, ut puta DXXV; per indictionem tertiam et annorum qui fuerint quartam partem semper adjice, id est, nunc CXXXI, qui simul fiunt DCLVI. His adde IV, fiunt DCLX. Hos partire per VII, remanent II. Duae sunt epactae solis, id est concurrentes septimanae dies, per suprascriptam indictionem, consulatu Probi junioris.	If you want to know the solar increments, that is the concurrent days of the week, add the years since the incarnation of the Lord that have passed, say 525; for the third indiction and the years that have passed until then always add the fourth part, which is now 131, these yield 656 altogether. To these add 4, yielding 660. Divide those by 7, 2 are left over. Two are the epacts of the sun, that is, the concurrent days of the week, for the indiction described above, in the consulship of Probus Junior.

For year number Y, this is intended to give concurrentes(Y) = 1 + (3 + Y + floor(Y / 4)) mod 7 as is confirmed by column four of the table above.

With the numbering of [Argumentum 12] for the days of the week (but with 7 instead of 0 for Saturday), this amounts to concurrentes(Y) = day of the week(Julian date(Y, March, 24)) which agrees with the concurrents for year number Y as defined by Bede about 200 years later.

Argumentum V. De cyclo decemnovennali.	Argumentum 5. On the cycle of nineteen years.
Si vis scire quotus sit annus circuli X et IX annorum, sume annos Domini, ut puta, DXXV, et unum semper adjice, fiunt DXXVI. Hos partire per X et IX, remanent XIII. Tertius decimus est annus cycli	If you want to know which year it is in the circle of 10 plus 9 years, add the years of the Lord, say 525, and always add one, yielding 526. Divide those by 10 plus 9, 13 are left over. The year is the thirteenth in

decemnovennalis. Quod si nihil remanserit, IX decima est.	the nineteen year cycle. If nothing would be left over, it is the 9teenth.

Thus, for year number Y, cycle of nineteen years(Y) = 1 + (Y mod 19), which is also known as the Numerus Aureus of the year. It is used only in [Argumentum 14].

<u>Argumentum VI. De cyclo lunari.</u> **Si vis scire quotus cyclus lunae est, qui decemnovennali circulo continetur, sume annos Domini, ut puta, DXXV, et subtrahe semper II, et remanent DXXIII. Hos partire per X et IX, remanent X. Decimus cyclus lunae est decemnovennalis circuli. Quoties autem nihil remanet, nonus decimus est.**	<u>Argumentum 6. On the lunar cycle.</u> If you want to know which cycle of the moon it is, that is contained in the nineteen year circle, add the years of the Lord, say 525, and always subtract 2, and 523 are left over. Divide those by 10 plus 9, 10 are left over. It is the tenth lunar cycle in the nineteen year circle. And whenever nothing is left over, it is the nineteenth.

Thus, for year number Y, lunar cycle(Y) = 1 + (Y - 3)mod 19, which is also known as the Jewish lunar cycle number machzor. Besides (Y mod 19) as used in [Argumentum 3] and the "cycle of nineteen years" of [Argumentum 5], it is the third function essentially equivalent to (Y mod 19). It is used only in [Argumentum 13] to compute a kind of Alexandrian epacts.

<u>Argumentum VII. De luna decima quarta in mense Martio.</u> **Si vis nosse quibus annis decemnovennalis circuli Martio mense, XIV luna paschalis incurrat: anno II, V, VII, X, XIII, XVI, XVIII, hos suprascriptos VII annos in Martio mense reperies; residuos vero XII, secundum regulam subter annexam, Aprili mense indubitanter calculabis.**	<u>Argumentum 7. On the fourteenth moon in the month of March.</u> If you want to find out in which years of the nineteen year circle the 14th paschal moon occurs in the month of March: in the year 2, 5, 7, 10, 13, 16, 18, in these 7 years above you shall see it in the month of March; but in the remaining 12 you will calculate it without doubt in the month of April, according to the rule appended below.

These are in fact all the numbers Y of years in which the age of the moon computed with the rule in [Argumentum 9] is 14 on some day from March 21 to March 31 (with (Y + 9)mod 19 mod 8 mod 3 = 0). The referenced rule probably is the one in [Argumentum 9] for April.

<u>Argumentum VIII. De bissexto.</u> **Si vis scire quando bissextus dies sit, sume annos Domini, ut puta DXXV. Partire hos per IV. Si nihil remanserit, bissextus est. Si I aut II, vel III, remanent, bissextus non est.**	<u>Argumentum 8. On the leap day.</u> If you want to know when the leap day is, add the years of the Lord, say 525. Divide those by 4. If nothing should be left over, there is a leap day. If 1 or 2 or 3 are left over, there is no leap day.

This says that Y is the number of a leap year iff Y mod 4 = 0.

Ne tibi forsitan aliqua caligo erroris occurrat, per omnem computum per quem ducis, si nihil superfuerit, eumdem computum esse per quem ducis agnosce, ut puta, si per X et IX ducis, et nihil superfuerit, XIX esse; si per XV, quindecimum, et, si per VII, septimum.	So that any unclarity does not possibly lead you into error, for all divisions you do, if nothing is left over, you should consider this computation to yield that by which you divide, thus for instance, if you divide by 10 plus 9, and nothing would remain, you should consider it to be 19; if by 15, then fifteen, and if by 7, then seven.

This rule says that the remainder of(A)upon division by(B) = 1 + (A - 1)mod B rather than just A mod B. This rule, however, is not always applied: (a) The remainder operations by 19 and by 30 in [Argumentum 3] and [Argumentum 11] must yield 0, so that, for Y mod 19 = 0, the epact is 0 (as asserted in [Argumentum 14] and the table above) and not 29; (b) in [Argumentum 12], the remainder upon division by 7 can be "nihil".

<u>Argumentum IX. De luna paschali mense Martio.</u> **Si vis cognoscere quota luna festi paschalis occurrat; si Martio mense Pascha celebratur, computa menses a Septembri usque ad Februarium, fiunt VI. His semper adjice regulares II, fiunt VIII; adde epactas, id est adjectiones lunares cujus volueris anni, ut puta, indictionis tertiae XII, fiunt XX; et diem mensis qua Pascha celebratur, id est Martii XXX, fiunt simul L. Deduc XXX, remanent XX; vicesima est in die resurrectionis Domini.**	<u>Argumentum 9. On the Easter moon in the month of March.</u> If you want to learn which moon it is on which the feast of Easter occurs; if Easter is celebrated in the month of March, compute the months from September to February, yielding 6. To this always add the correction 2, yielding 8; add the epacts, that is, the lunar increments of the year you want, say 12 for the third indiction, yielding 20; and the day of the month on which Easter is celebrated, that is March 30, yielding together 50. Deduct 30, 20 are left over; the twentieth [moon] is on the day of the resurrection of the Lord.

*This amounts to age of the moon on(Julian date(Y, March, D)) = ((Y mod 19)*11 + 6 + 2 + D)mod 30 = (age of the moon on(Julian date(Y, March, 22)) - 22 + D)mod 30 if Easter is Julian date(Y, March, D). But of course it works for any day number D between 22 and 31, and for all year numbers Y, not just those of [Argumentum 7]. The year number for the example could be 0525.*

In this calculation and the following one for dates in April, Dionysius suggests that the epacts for year Y not only give the age of the moon at March 22, as stated in [Argumentum 11], but also at some day around September of year (Y - 1). Only late August and late September would work: Julian date(Y, March, 22) ~= 7 synodic month + Julian date(Y - 1, August, 27.29 or 28.29) ~= 6 synodic month + Julian date(Y - 1, September, 25.82 or 26.82) (where the second day numbers are to be taken iff Y is divisible by 4).

Mense Aprili. - **Si vero mense Aprili Pascha celebramus, computa menses a Septembri usque ad Martium, fiunt VII. His semper adjice II, fiunt IX. Adde epactas lunae anni cujus volueris, ut puta, indictionis IV, XXIII, qui fiunt XXXII, et diem mensis quo Pascha celebramus, id est Aprilis XIX, qui simul fiunt LI; deduc XXX, remanent XXI. Luna XXI est in die resurrectionis Domini.**	In the month of April. - If however we celebrate Easter in the month of April, compute the months from September to March, yielding 7. To this always add 2, yielding 9. Add the lunar epacts of the year you want, say 23 for indiction 4, yielding 32, and the day of the month in which we celebrate Easter, that is April 19, which together yield 51; deduct 30, 21 are left over. The age of the moon is 21 on the day of the resurrection of the Lord.

This amounts to the same formula as above for the remaining year numbers: age of the moon on(Julian date(Y, April, D)) = (age of the moon on(Julian date(Y, March, 22)) + 9 + D)mod 30 Thus, the age of the moon is supposed to increase by 1 for each day throughout the 35 day interval from March 21 until April 25 in which these formulae are applicable; this agrees with columns 6 and 8 in the table above. The year number for the example could be 0526.

Si requiras a Septembri usque ad Decembrem, tres semper in his IV mensibus regulares adjicias: in bissexto autem solummodo anno duos regulares suprascriptis mensibus adnumerabis, et pro XXXI	If you need it from September to December, you should always add the correction three in these 4 months: only in a leap year you also shall add the correction two for these months described above, and

die, XXXII annis singulis Decembri mense assumes in fine.	finally in non-leap years, for day 31 in the month of December you should assume 32.

*This is probably meant as a recipe similar to the two above for the age of the moon on(Julian date(Y, January, D)) = ((Y mod 19)*11 + 4 + 3 + (1 or 2) + D)mod 30 where the "4" acts as the number of months from September to December, "3" is the the correction in every year, and the "(1 or 2)" comes either from the correction 2 for leap years, or, for non-leap years, it is an interpretation of the effect of assuming 32 days in December.*

The interpretation above is consistent with [Argumentum 11] since: Julian date(Y, January, 00) ~= Julian date(Y, March, 22) - 3 synodic month + (7.6 or 8.6) d (with 8.6 instead of 7.6 for leap year numbers Y).

<u>Argumentum X. De die septimanae sanctae feria paschali.</u> **Si vis cognoscere quotus dies septimanae est, sume dies a Januario usque ad mensem quem volueris, ut puta, ad XXX diem mensis Martii, fiunt LXXXIX. His adjicies semper unum, fiunt XC; et semper adde epactas solis, id est concurrentes septimanae dies cujus volueris anni, ut puta II, indictionis III, fiunt simul XCII. Hos partire per VII, remanet una: ipsa est dominica paschalis festi. Sic quamlibet diem a calendis Januarii usque ad XXXI diem mensis Decembris, quota feria fuerit, invenies computando, ut regularem unum et concurrentes, quae a Januario mense semper incipiunt, pariter assumas.**	<u>Argumentum 10. On the day of the holy week of the feast of Easter.</u> If you want to learn which day of the week it is, add the days since January until the month you want, say until March 30, there are 89. To this always add one, yielding 90; and always add the solar epacts, that is, the concurrents of the seven day week for the year you want, say 2 for the indiction 3, yielding 92 altogether. Divide those by 7, one is left over: this is the Sunday of the feast of Easter. In this way, if you venture to compute which day of the week it is for any day from the first of January until the 31st of the month of December, you should equally assume the correction one and the concurrents which always begin in the month of January.

The example date could be Julian date(0525, March, 30), as can be seen from the table above. The example shows that the number of days from January to(Julian date(Y, January, 01) + D d) is meant to be D + 1 rather than D ("Roman inclusive counting"). With the solar epacts of [Argumentum 4], the formula given amounts to day of the week number(Julian date(Y, January, 01) + D d) = (D + 1 + 1 + 4 + Y + floor(Y / 4))mod 7 = (D + (Y - 1) + floor(Y / 4))mod 7 which agrees with the correct formula of [Argumentum 12] only if Y is not divisible by 4, and otherwise is one day ahead.

<u>Argumentum XI. De luna citimi paschalis.</u> **Si vis scire quota luna sit in XI calendas Aprilis, sume annos incarnationis Domini nostri Jesu Christi, ut puta, DCLXXV. Hos partire per [*XIX, remanent X; et multiplica decem per*] XI, fiunt CX. Partire tricesima, remanent XX: vicesima luna est in XI calendas Aprilis. Si autem VII, septima; si asse, prima.**	Argumentum 11. On the moon closest to Easter. If you want to know which moon it is on March 22, add the years since the incarnation of our Lord Jesus Christ, say 675. Divide those by [*19, 10 are left over; and multiply ten by*] 11, yielding 110. Divide by 30, 20 are left over: it is the twentieth day of the moon on March 22. And if 7 [is left over], then the seventh, if one, the first.

*Only with the suggested correction (and allowing for remainders of zero) this yields age of the moon in year(Y)on March 22 = ((Y mod 19)*11) mod 30 which are the epacts of [Argumentum 3]. Thus, Dionysius Exiguus uses Julian date(Y, March, 22) as "sedes epactorum" (seating of the epacts).*

*Besides these so-called Dionysian epacts, several other epacts have been used in computs for the same or a different Easter date, such as the Alexandrian epacts (8 + (Y mod 19)*11) mod 30, which, according to [Argumentum 13], would give the nominal age of the moon on the day preceding January 01.*

<u>Argumentum XII.</u> **Si vis nosse diem calendarum Januarii, per singulos annos, quota sit feria, sume annos incarnationis Domini nostri Jesu Christi, ut puta, annos DCLXXV. Deduc assem, remanent DCLXXIV. Hos per quartam partem partiris, et quartam partem, quam partitus es, adjicies super DCLXXIV, fiunt simul DCCCXLII. Hos partiris per VII, remanent II. Secunda est dies calendarum Januarii. Si V, quinta feria; si asse, dominica; si nihil, sabbatum.**	<u>Argumentum 12.</u> If you want to find out which day of the week it is on the first day of January, for non-leap years, then add the years since the incarnation of our Lord Jesus Christ, say 675 years. Subtract one, 674 are left over. Divide those into the fourth part, and add the fourth part obtained by the division to 674, yielding 842 altogether. Divide those by 7, 2 are left over. It is Monday on the first of January. If 5 [are left over] then [it is] Thursday, if one, then Sunday; if nothing, Saturday.

This amounts to day of the week number of year (Y) on January 01 = ((Y - 1) + floor((Y - 1)/4))mod 7 with 0 for Saturday, which is in fact the number for day of the week (Julian date(Y, January, 01)). This is true for leap year numbers Y as well.

<u>Argumentum XIII. De luna calendarum Januarii.</u> **Si vis scire quota luna sit calendis Januarii, scito quotus lunaris cyclus sit, verbi gratia cyclus XV. Tene tibi unum, id est ipsas calendas Januarii, et duces quinquies quinquies decies: faciunt LXXV; quos adjicies super unum, et fiunt LXXVI. Item duces sexies decies quinquies, faciunt XC; quos adjicies super LXXVI, et sic summa numerorum CLXVI; in quibus partiris tricesima, remanent XVI. Sexta decima luna est calendis Januarii, et puncti XVI. Isto modo per XIX cyclos lunares computabis semper, et calendis Januarii, quota sit luna, absque errore reperies.**	<u>Argumentum 13. On the age of the moon on the first of January.</u> If you want to know which moon it is on January 01, knowing which lunar cycle it is, for instance cycle 15. Retain one, which is for the same January 01, and take five fifteen times: yielding 75; to which you always add one, thus yielding 76. Now take six fifteen times, making 90, which you add to 76, thus the sum of the numbers is 166; divide these into the thirtieth [part], 16 are left over. It is the sixteenth moon on January 01, and 16 puncti. In this way you can always compute for the 19 cycles of the moon, and you will obtain without error the age of the moon on January 01.

*For year numbers Y, and with the lunar cycle L = 1 + (-3 + Y)mod 19 from [Argumentum 6], this computation yields age of the moon on(Julian date(Y, January, 01)) = (L*5 + 1 + L*6)mod 30 = (12 + ((-3 + Y)mod 19)*11)mod 30 =(for Y mod 19 >= 3:) (9 + (Y mod 19)*11)mod 30 up to to the puncti (to be discussed below).*

Unless Y is divisible by 4, this agrees with the formula suggested at the end of [Argumentum 9].

Dum autem veneris ad XVII cycli lunaris, et duxeris quinquies decies septies, super calendas Januarii, qui faciunt LXXXV, si partiris sexagesima, et adjicies ipsum assem, fiunt LXXXVI. Deinde ducis sexies decies septies, fiunt CII. Eos adjicies super LXXXVI, et fiunt CLXXXVIII. [*Adiicies unum, fiunt CLXXXVIIII.*] Partire ibi tricesima, remanent IX. Nona luna est calendis Januarii, et puncti XXVI. Sic et in XVIII et XIX cyclo facies. A primo vero cyclo lunari, usque in sextum decimum, non partiris sexagesimam, ne in errorem incidas.	As soon as you shall come to lunar cycle 17, then take five times seventeen, after January 01, which makes 85, if you divide into the sixtieth [part], and add the resulting one to it, this yields 86. Meanwhile take six times seventeen, yielding 102. Those add to 86, and it yields 188. [*Add one, yielding 189*]. Divide this by thirty, 9 are left over. It is the ninth moon on January 01, and 26 puncti. In this way you also compute in cycles 18 and 19. From the first lunar cycle until the sixteenth you do not divide by 60 so as not to make an error.

For the Julian year number Y, this computation is said to apply if L = 1 + (-3 + Y)mod 19 is 17, 18, or 19, that is, if L = Y mod 19 + 17. With the addition of 1 as amended above in brackets, it yields age of the moon on(Julian date(Y, January, 01)) = (L*5 + floor(L/12) + L*6 + 1) mod 30 =(for Y mod 19 < 3:) (9 + (Y mod 19)*11)mod 30 resulting in the same formula as above for the remaining year numbers Y.

Apparently, a separate formula is given for 17 <= L <= 19 because of the term floor(L/12). Of course, floor(L/17) would have worked for all L; this would have required the remainder modulo 85 instead of modulo 60. With the 19 year cycle (as in [Argumentum 5]), a single (and simpler) formula would do.

The separate multiplication by 5 in both computations above is very likely due to a formula of the type fractional age of the moon on(Julian date(Y, January, 01)) = (A + (Y - B)*30*235/19)mod 30 = (A + ((Y - B) mod 19)*(5*(1 + 1/95) + 6)) mod 30 derived directly from the Metonic value for the synodic month. For integral B and suitable A it gives values for the age of the moon that are integral multiples of 1/95. The number 1/95 is close to a 1/96 = (1 punctus)/(1 d) (see [Argumentum 16]) which would explain the appearance of puncti in the age of the moon.

Unfortunately, the text is not explicit about the computation of the puncti, and the two examples leave many possibilities open, such as: age of the moon on(Julian date(Y, January, 01))in days and puncti = (11 + 42/96 + ((Y - 3) mod 19)*(5*(1 + 1/96) + 6)) mod 30 or = (37/96 + ((Y - 2) mod 19)*(5*(1 + 1/96) + 6)) mod 30 or = (19 + 32/96 + ((Y - 1) mod 19)*(5*(1 + 1/96) + 6)) mod 30 And if we assume that the second example is meant to yield an age of the moon of 8 (rather than 9) plus 16 puncti, then we could have age of the moon on(Julian date(Y, January, 01))in days and puncti = (8 + 27/96 + (Y mod 19)*(5*(1 + 1/96) + 6)) mod 30 In all these formulae, the age of the moon increases by 11 + 5/96 per year except for the "saltus" of 11 + 6/96 once every 19 years.

Thus, this Argumentum incompletely describes a kind of Alexandrian epacts that apparently already had been described more fully elsewhere; I do not know such a source, however.

<u>Argumentum XIV. Quota feria luna XIV incidat cycli decemnovennalis anno primo.</u> **Incipit calculatio quomodo reperiri possit quota feria singularis anni decima quarta luna paschalis, id est primi circuli decemnovennalis.**	<u>Argumentum 14. On which day of the week the fourteenth moon falls in the first year of the nineteen year cycle.</u> The calculation begins whereby one can find out on which day of the week the fourteenth paschal moon falls in a single year, this one being for the first circle of nineteen.
Anno primo, quia non habet epactas lunares, pro eo quod cum noni decimi inferioris anni XVIII, et suis XI epactis, addito etiam ab ¦gyptiis die una, fiunt XXX, id est luna mensis unius integra, et nihil remanet de epactis, et quod in Aprili mense incidit eo anno luna paschalis XIV, tene regulares in eo semper XXXV, subtrahe XXX, id est ipsa luna integra, et remanent V. Quinto die a calendis, hoc est nonis Aprilis, occurrit luna paschalis XIV. Tene suprascriptos V, adde et concurrentes ejusdem anni IV, fiunt IX. Adde et regulares in eodem semper mense Aprili VII, fiunt XVI. Hos partire per VII, id est bis septeni XIV, remanent II. Secunda feria occurrit luna paschalis XIV, et dominicus festi paschalis dies luna XX.	In the first year, which does not have lunar epacts, because to those 18 from the previous nineteenth year, and its 11 epacts, one day is added by the Egyptians, yielding 30, that is one full lunar month, so that nothing remains from the epacts, and so that in this year the 14th paschal moon falls in the month of April, in this year always take the correction 35, subtract 30, that is this full month, and 5 remains. The 14th paschal moon occurs five days from the Kalends, which is April 05. Take the 5 from above, and add the concurrents 4 of this year, yielding 9. And always add to this the correction 7 in the month of April, yielding 16. Divide those by 7, that is, two times seven is 14, 2 are left over. On Monday occurs the 14th paschal moon, and the Sunday of the Easter holiday on the day of the 20th moon.

For Julian year number Y, the rule first given is meant to be day of the 14th paschal moon in year(Y) = April 35 - (16 + (-16 + (Y mod 19)*11)mod 30) d for the special case Y mod 19 = 0. For all integral Y, this is equal to March 21 + (15 - (Y mod 19)*11)mod 30 d which is in fact the first date >= Julian date(Y, March, 21) whose age of the moon is 14 according to [Argumentum 11], and using an increase in the age of the moon of 1 mod 30 per day. With the numbering of the days of the week as in [Argumentum 12], the second rule is: (35 - 16 - (-16 + (Y mod 19)*11)mod 30 + day of the week(Julian date(Y, March, 24) + 7)mod 7 = (day of the week(Julian date(Y, March, 24) - 3 + (15 - (Y mod 19)*11)mod 30)mod 7 = day of the week(day of the 14th paschal moon in year(Y)) The addition of 7 in this rule is of course unnecessary; it just ensures that X mod 7 is never evaluated with X < 7.

The first sentence describes the "saltus lunae" when the epacts increase by 12 rather than 11 from year (Y - 1) to year Y with Y mod 19 = 0.

Anno secundo. Item praefati circuli annus secundus, a quo sumunt exordium epactae XI. Incidit in eo anno luna paschalis XIV mense Martii. Tene XXXVI regulares in eo semper, subtrahe semper epactas XI, remanent XXV. Vicesimo quinto die a calendis Martii, quod est VIII calendas Aprilis, occurrit luna paschalis XIV. Tene suprascriptos XXV, adde concurrentes ejusdem anni V, fiunt XXX. Adde semper in fine hujus mensis regulares IV, hos partire per VII, id est septies quaterni XXVIII, remanent VI. Sexta feria occurrit luna XIV paschalis, et dominicus festi paschalis dies luna XVI.	In the second year. Now to the second year of the above mentioned circle, for which the epacts add up to 11 to begin with. In this year, the 14th paschal moon occurs in the month of March. In this [month], always take the correction 36, always subtract the epacts 11, 25 are left over. The 14th paschal moon occurs twenty five days from the beginning of March, that is, on March 25. Take the 25 from above, add the concurrents 5 for this year, yielding 30. Finally always add the correction 4 for this month, divide those by 7, that is four times seven or 28, 6 remain. The 14th paschal moon occurs on Friday, and the Sunday of the feast of Easter is the day of the 16th moon.

For Julian year number Y, the rule given first is day of the 14th paschal moon in year(Y) = March 36 - ((Y mod 19)*11)mod 30 d and if the 14th mooon is in March, then this is again equal to March 21 + (15 - (Y mod 19)*11)mod 30 d because ((Y mod 19)*11)mod 30 <= 15 in this case (see [Argumentum 7]). The second rule also amounts to the same as above.

Anno tertio. - Item mense Aprili saepe dicti circuli primi anno tertio. Tene semper in eo mense imprimis regulares XXXV. Subtrahe epactas ejusdem anni XXII, remanent XIII. Tertio decimo die mensis, id est idibus Aprilis, occurrit luna paschalis XIV. Tene hos XIII, adde concurrentes VI, fiunt XIX. Adde in Aprili semper inferius regulares VII, fiunt XXVI. Hos partire per VII ter septeni, XXI, remanent quinque. Quinta feria erit decima quarta luna paschalis, et dominicus dies paschalis festi luna XVII.	In the third year. In the third year of said first cycle, [the 14th moon occurs] also always in the month of April. For this month, always take first the correction 35. Subtract the epacts 22 of this year, 13 are left over. The 14th paschal moon occurs on the thirteenth day of the month, that is on April 13. Take those 13, add the concurrents 6, yielding 19. Then always add in April the correction 7, yielding 26. Divide those by 7, three times 7 [are] 21, five are left over. On Thursday was the fourteenth paschal moon, and the Sunday of the feast of Easter on the 17th moon.

These are the same rules as for the first year.

Ita singulis annis a primo usque ad nonagesimum quintum annum calculabis. Si quando mense Martio XIV luna paschalis incurrit, XXXVI regulares imprimis teneas, ex quibus epactas cujus volueris anni deducas, et concurrentes adjicias, et in fine: semper IV regulares augmentes. Aprili vero mense semper XXXV in capite tene, ex quibus, ut supradictas epactas, et adjectos ejusdem anni concurrentibus suis regulares in fine VII augmenta. Facilius namque et brevius omnia argumenta paschalia calculabis. Hoc tamen praeterea lectori sit cognitum, quoties in utrosque menses suprascriptos in prima regula contigerit, ut deductas epactas, amplius a XXX remaneant, dimitte XXX. Quod si unus aut duo, vel amplius superfuerint, tot dies ipsius mensis a calendis Januarii [*? Aprilis*] sit luna paschalis XIV. Quando autem (post) deductas epactas infra XXX [? XXI] ut puta XX, seu amplius minusve remanserit, quod semel in XIX annis accidere manifestum est, XXX die Aprilis erit luna paschalis XIV.	In this way you calculate for each year from the first to the nineteenth year. When the 14th moon occurs in the month of March, then you first take the correction 36, from which you deduct the epacts of the year you want, and add the concurrents, and finally you always add the correction 4. But in April you keep 35 in mind, from which [you take] the epacts mentioned above, and finally add the correction 7 increased by the concurrents of the same year. Thus you will calculate all the argumenta for Easter more easily and faster. Above all, let the reader know that, whenever it happens that more than 30 are left over when the epacts are deducted for any of the months described above to which the first rule applies, then dismiss 30. When one or two or more are left over, then so many days from January 01 [*? April 01*] is the 14th paschal moon. And if less than 30 [*? 21*] should be left over (after) the epacts have been deducted, say 20, or more or less, which is bound to happen once in 19 years then the 14th paschal moon will be on the 30th day of April.

The first part repeats the rules which have already been applied in the preceding paragraphs to the cases where Y mod 19 is <= 2.

*The last portion of the text contains several errors and seems to deal with the case when the "first" formula March 36 - ((Y mod 19)*11)mod 30 d is applied when the 14th paschal moon is in April, in which case it yields a date after March 31 or (a wrong one) before March 21. Thus, if 36 - ((Y mod 19)*11)mod 30 is > 30 then it is also > 31 and the 14th paschal moon is on April 00 + (5 - ((Y mod 19)*11)mod 30) d rather than on April 00 + (6 - ((Y mod 19)*11)mod 30) d as asserted in the text. And if 36 - ((Y mod 19)*11)mod 30 is < 21 then the 14th paschal moon is again in April. Note that the example where it is supposed that 36 - ((Y mod 19)*11)mod 30 equals 20 cannot occur (because ((Y mod 19)*11)mod 30 is never 16 for integral Y). The largest values < 21 that can occur are 19 (for Y mod 19 = 7) and 18 (for Y mod 19 = 18). And, of course, a 14th paschal moon never is on April 30.*

<u>Argumentum XV. De die aequinoctii et solstitii.</u> **Qua die natus est Dominus Jesus Christus secundum carnem ex Maria Virgine in Bethlehem, in qua incipit crescere dies. Aequinoctium primum est in VIII calendas Aprilis, in qua aequatur dies cum nocte. Eodem die Gabriel nuntiat sanctae Mariae, dicens: Spiritus sanctus superveniet in te, et virtus altissimi obumbrabit te. Propterea quod ex te nascetur, vocabitur Filius Dei. In qua etiam passus est Christus secundum carnem. Solstitium secundum est VIII calendas Julii, quando etiam natus est sanctus Joannes Baptista ex quo incipit decrescere dies. Aequinoctium secundum est VIII calendas Octobris, in qua die conceptus est Joannes Baptista. Et hinc jam minor efficitur dies nocte, usque ad natalem Domini Salvatoris. Ex VIII calendas Aprilis et in VIII calendas Januarii, dies numerantur CCLXXI. Unde**	<u>Argumentum 15. On the day of the equinox and the solstice.</u> The day on which the Lord Jesus Christ was born into flesh from the Virgin Mary in Bethlehem is the one on which the day begins to increase. The first equinox is on March 25, when day is equal with night. On this very day Gabriel annunciates to Holy Mary, saying: The Holy Ghost shall come upon thee, and the power of the Highest shall overshadow thee. Therefore also that which shall be born of thee shall be called the Son of God. [Luke 1.35, courtesy King James] Also on this day Christ has suffered in the flesh. The second solstice is on June 24, from which the day starts to decrease, and also when Saint John the Baptist was born. The second equinox is on September 24, on which day John the Baptist was conceived. And right from then on until the birth of

secundum numerum dierum conceptus est Christus Dominus noster in die dominica VIII calendas Aprilis, et natus est in III feria XIII calendas Januarii Christus Dominus noster. In die qua passus est, fiunt anni CXXXIII [*? XXXIII*] et menses III, qui sunt dies XII CCCCXIIII. Unde secundum numerum dierum ejus stat cum III feria natum, et passum VI feria: natum VIII calendas Januarii, passum VIII calendas Aprilis. Ex quo baptizatus est Jesus Christus Dominus noster, fiunt anni II, et dies numerantur XC, qui fiunt DCCCXX, cum bissextis diebus suis, ac sic baptizatur VIII idus Januarii die, V feria, et passus est, ut superius dixi, VIII calendas Aprilis, VI feria. Cum bissextis diebus suis fiunt simul dies XII CCCCXV, et (ab) VIII idus Januarii in VIII calendas Aprilis dies XC.	the Lord and Saviour, the day becomes shorter than the night. From March 25 and until December 25, the days number 271. And that number of days after our Lord Christ was conceived on Sunday March 25, our Lord Christ was born on Tuesday December 20. On the day on which he has suffered death, 133 [? 33] years and 3 months have elapsed, which are 12 [thousand] 414 days. And that number of days after his birth took place on a Tuesday, he suffered death on a Friday: he was born on December 25 and suffered death on March 25. From when our Lord Jesus Christ was baptized, there were 2 years and the days numbered 90, yielding 820, with its leap days, and so he was baptized on the day January 06, a Thursday, and suffered death, as I said above, on March 25, a Friday. With its leap days this yields 12 [thousand] 415 days altogheter, and 90 days (from) January 06 to March 25.

This Argumentum does not concern the determination of Easter but certain ecclesiastical dates connected with the life of Jesus. Moreover, the numbers and dates in the text of this Argumentum are not consistent with the rest of the liber. They are even inconsistent among themselves, and there is no obvious reading that would make them consistent. In fact, the inconsistencies are so easy to spot that we may assume that the author of this Argumentum was not even concerned with chronological correctness nor consistency with the preceding Argumenta. The rest of this comment indicates some of the inconsistencies.

*The date of birth of Jesus is given as December 25 several times, and once as December 20; there is also a reference to January 06 which is another popular date for nativity. The number of 271 days from conception to birth, as given in the text, would fit one of these, using "Roman inclusive counting": December 20 - preceding March 25 = 270 d = 38*7 d + 4 d but it does not fit December 25. On the other hand, if conception is on March 25 and on a Sunday, and birth is on a Tuesday, then birth has to be on December 25.*

*The next time interval mentioned is given as 12 414 d and as 12 415 d. One has 12 414 d = 1773*7 d + 3 d = 34*365.25 d - 4.5 d = Julian date(Y + 34, March, 21) - Julian date(Y, March, 25) or = Julian date(Y + 34, March, 20) - Julian date(Y, March, 25) depending on whether floor(Y/2) is even or odd. This could be a miswritten value (some 4 d too small) for the time interval from conception to death, but it certainly is not any integral number of years plus 3 months as pretended. Assuming a different scribal error, it could also be meant as the time interval from birth to death: 11 414 d = 1630*7 d + 4 d = 31*365.25 d + 91.25 d = Julian date(Y + 32, March, 25) - Julian date(Y, December, 25) or = Julian date(Y + 32, March, 25) - Julian date(Y, December, 24) depending on whether Y is divisible by 4 or not. This can be said to be 32 years (but not 33 years) plus 3 months. If 11 413 d were actually meant ("Roman inclusive counting"), this would even be compatible with the days of the week Friday and Tuesday for death and birth (the other reading would not).*

However, these days of the week are inconsistent with the numbering of years since the incarnation: the year numbers closest to 0 yielding a Sunday for March 25 are Julian date(-0003, March, 25) and Julian date(0003, March, 25) as can be seen easily from the table above and also from [Argumentum 4]. (We use the astronomical numbering of years .., -0001, 0000, +0001,.. for which the formula of [Argumentum 4] is always valid).

*The next time interval mentioned is 820 d = 117*7 d + 1 d = 2*365.25 d + 89.5 d = Julian date(Y + 3, March, 25) - Julian date(Y, December, 25) or = Julian date(Y + 3, March, 25) - Julian date(Y, December, 26) depending on whether or not Y is divisible by 4. While this could be considered as 2 years and 90 days "with its leap days", it is not consistent with the date January 06 for the baptism. 810 days would be consistent with that date but not with the day of the week Thursday for the baptism.*

*The last time interval mentioned is Julian date(Y, March, 25) - Julian date(Y, January, 06) = 79 d or = 78 d = 11*7 d + 1 d depending on whether Y is divisible by 4 or not; only in the latter case can the two dates be a Friday and a Thursday. This is incorrectly given as 90 d = 12*7 d + 6 d, which happens to be Julian date(Y, March, 25) - Julian date(Y - 1, December, 25) unless Y is divisible by 4.*

Using the Easter dates of the table above for year numbers around 0562 it is also easy to see that March 25 never was a Good Friday in the years with numbers around 0030; Julian date(0034, March, 26) is the closest.

<u>Argumentum XVI. De ratione bissexti.</u> **Bissextum non ob illum diem fieri, ut quidam putant, quo Josua oravit solem stare, credendum est: quia dies ille et fuit, et praeteriit. Sed ab hoc dicitur bissextus, quod in unumquemque mensem punctus unus accrescit. Punctus vero unus quarta pars horae est. IV vero puncti unam horam faciunt; XII vero puncti III horas explicant. Ergo in VI annis ternae horae, quae sunt XII, diem faciunt I, qui addatur Februario, cum VI calendas Martii habuerit, ut in crastino sic habeat. Verbi causa, si hodie VI calendas Martii additur ille dies in IV anno expleto; nihilominus et crastino VI calendas Martii habeatur. Et ideo bissextus dicitur, quia bis VI calendas Martii habet Februarius.**	<u>Argumentum 16. On the rationale of the leap day.</u> One must not believe what some people maintain, that the leap day has arisen from that day on which Joshua commanded the sun to stand still: that day has been and is long gone. But it is called leap day because it gains one punctus in each month. The punctus is indeed the fourth part of an hour. And 4 puncti make one hour; and 12 puncti explain 3 hours. Hence in 4 years three hours each, which are 12, making 1 day which is added to February, so that when it is February 24, it is the same the next day. For instance, if today is February 24 and that day is added if 4 years are complete; then it will nevertheless be February 24 tomorrow. And it is called bisextile because February has two times the 6th of the calends of March.

*In this "explanation", a leap day accumulates from 1/48 d per month. Because 1 d is taken to be 12 h, the 1/48 d per month is taken to be 1 punctus = 1/4 h = 1/96 d = 1/48*12 h per month.*

Sex diebus fecit Deus mundum, septimo requievit. Ut ergo plenius intelligatur, computa quot horas habeat unus dies [*? annus*], et divides illas in VII partes, et quantus remanet, exinde sit bissextus. Primum computa dies CCC, quomodo horas habent, decies tricenteni sunt tria millia. Iterum facis: bis tricenteni, sexcenteni: fiunt in tricentis diebus horae III DC. Iterum facis: decies sexageni DC, et bis sexageni CXX. Fiunt ergo in sexagenis diebus horae DCXX [DCCXX]. Iterum facis: decies quini L, et bis quini X. Ecce habes in quinque diebus horas LX. Fiunt simul integro anno in diebus CCCLXV horae IIII CCCLXXX, et alias tantas in nocte, fiunt simul dierum et noctium totius anni VIII DCCLX horae. Divide in illas VII partes. Primum facis: septies milleni VII, remanent I DCCLX. Item facis: septies ducenti, fiunt I CCCC, remanent CCCLX. Item facis: septies quinquageni, fiunt CCCL, remanent X. Item facis: septies as VII, remanent III. Istae tres horae faciunt in IV annis diem.	In six days God created the world, on the seventh he rested. So that this can be more fully understood, compute the number of hours one day [? year] has, and divide those into 7 parts, and the leap day shall come from what is left over. First compute how many hours 300 days have, ten times three hundred are three thousand. Then do: two times three hundred, six hundred: yielding 3600 hours in three hundred days. Then do: ten times six [is] 60, and two times sixty [is] 120. Thus, this yields 620 [720] hours in sixty days. Then do: ten five times [is] 50, and two times five [is] 10. Thus you have 60 hours in five days. Together, a whole year in 365 days yields 4 [thousand] 380 hours, and as many also in the night, yielding with day and night together 8760 hours. Divide those into 7 parts. First do: seven times thousand [is] 7[000], 1 [thousand] 760 are left over. Then do: seven times two hundred yield 1400, 360 are left over. Then do: seven times fifty yield 350, 10 are left over. Then do: seven times one [is] 7, 3 are left over. These three hours make a day in 4 years.

Here, a leap day accumulates from 1/4 d per year. And 1/4 d per year is "explained" with numerology: 1/4 d is taken to be 3 h (assuming that 1 d is 12 h) and explained as (365 d) mod (7 h) = (8760 h) mod (7 h) = 3 h which is correct only if we assume that 1 d is 24 h.
